Impact of Logistic Processes on Economic Order Quantity with Quantity Discount: An Optimization Approach

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ABSTRACT

Inventory processes have significant importance in a company’s logistics system and greatly influences its economic operation. Inventory management systems are used in the case of both dependent and independent products. The most used inventory model is the computation of economic order quantity model, which can be used in various types of objective functions and constraints. Logistic processes and their materials handling operations have a great impact on the optimal solution of economic order quantity problems, therefore it is important to consider logistic aspects while using EOQ. Within the frame of this article the authors describe a model considering available storage and transportation capacity, while fixed order lots as size of loading units and quantity discounts are taken into consideration. Two scenarios are discussed to validate the model and highlight the importance of logistics related constraints in computation of economic order quantity.

Keywords: Economic order quantity; logistic cost; transportation; quantity discount.
1. INTRODUCTION

Purchasing logistics is a subsystem of the integrated logistics chain, whose task is to prepare the raw materials, parts and semi-finished products required for production and the relevant information according to the needs of production. One of the main functional areas of procurement is purchasing, which can have a strong impact on the entire logistics process. This is because the purchase ensures the availability of supplier capacities in production. A key logistics task on the purchasing side is also to optimize inventories and involve suppliers in the corporate logistics process. Procurement logistics requires a number of strategic decisions that are essential for its proper functioning: how to select suppliers, making the decision to manufacture or buy, selection of delivery method (traditional, just-in-time, just-in-sequence, etc.), selection of IT solutions and standards, delivery organization, decisions related to the receipt of goods. The purchasing processes cover both dependent and independent products. An independent demand is a product if the demand for it does not depend on the demand for other products. When creating inventory management models for independent demand products, two important questions need to be answered: when and how much to order. There are, of course, many possible answers to this question, some possible models and approaches are discussed in Table 1.

Table 1. Economic order quantity related research and their topics

<table>
<thead>
<tr>
<th>Articles and topics</th>
<th>delay in payment</th>
<th>inspection/quality policy</th>
<th>multi-item</th>
<th>allowable rework</th>
<th>stock level</th>
<th>Fuzzy system</th>
<th>shortage</th>
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In our previous studies, we were focusing on the context between the shift of average demand and the safety stock of purchased parts [25], while the effect of the safety stock on the occurrence probability of the stock shortage was also considered [26]. Based on the results of these predecessor studies and the above-mentioned literature background, the article is organized as follows. Chapter 2 describes the used notations of the mathematical model of economic order quantity problem with volume discount and logistic aspects. Chapter 3 presents the mathematical model of an economic order quantity problem, where warehouse capacity, transportation capacity and order lot as loading unit size are considered. Chapter 4 demonstrates the results of computational studies and validates the model. Conclusions and managerial impacts are discussed in the remaining part of the article.

2. NOTATIONS

Throughout the article, the following notations are used to describe the mathematical model of economic order quantity problem with volume discount and logistic aspects:

- $C$ is the total cost of the time window of the analysis,
- $P$ is the total purchasing price,
- $C_w$ is the warehousing cost of the purchased products stored in a warehouse between order and manufacturing,
- $C_o$ is the cost of orders,
- $p(q)$ is the specific purchasing price, which is a function of order quantity,
- $Q$ is the total demand of the product for the whole time-window,
- $c_w$ is the specific warehousing cost in \( \text{USD/day-pcs} \),
- $q$ is the order quantity,
- $c_o$ is the specific order cost in \( \text{USD/order} \),
- $c_{MH}$ is the materials handling cost of the purchased products from the suppliers to the manufacturer,
- $p(q_i)$ is the specific purchasing price of product $i$,
- $Q_i$ is the total demand of the product $i$ for the whole time-window,
- $\Gamma$ is the set of orders, which are stored in the available warehouse,
- $\Psi$ is the set of orders, which are stored in the outsourcing warehouse,
- $\varnothing$ is the multiplier of the outsourced specific warehousing cost,
- $\text{CAP}_{WH}$ is the available warehouse capacity without outsourcing opportunities,
- $c_{oi}$ is the specific order cost of product $i$,
- $c_{mini}$ is the specific material handling cost of product $i$.

3. MATHEMATICAL MODEL

The conventional computing method of economic order quantity with quantity discounts is extensively discussed in many sources [27,28]. This single-item economic order quantity model is based on the concept that the suppliers offer price discount depending on the ordered quantity. In this case the optimization model of the economic order quantity is based on the cost related objective function:

$$C = P + C_w + C_o \rightarrow \text{min.}$$ (1)

The first part of the objective function is the purchasing price, which depends on the specific purchasing price and the purchasing quantity:

$$P = p(q) \cdot Q$$ (2)

The second part of the objective function is the warehousing cost, which depends on the specific warehousing cost and the order quantity:

$$C_w = c_w \cdot \frac{q}{2}$$ (3)

The third part of the objective function is the order cost, which depends on the specific order cost and the order quantity:

$$C_o = c_o \cdot \frac{Q}{q}$$ (4)

We can define the lower and upper quantity limit of quantity discount rates as $q_{0j}^{\min}$ and $q_{0j}^{\max}$ and the specific purchase price for this quantity range and we can write that

$$\forall q: p(q) = p_j(q) \text{if } q_{0j}^{\min} \leq q \leq q_{0j}^{\max}$$ (5)

Based on this above-mentioned simple methodology we can define the economic order quantity:

$$q_{opt} = q_j \rightarrow \text{min}_j(C(q_j))$$ (6)

Where

$$q_j = \begin{cases} q_{0j}^{\min} & \text{if } q_{0j}^{\min} \leq q_{opt} \leq q_{0j}^{\max} \\ q_{0j}^{\max} & \text{if } q_{0j}^{\min} \leq q_{opt} \leq q_{0j}^{\max} \\ q_{0j}^{\min} & \text{if } q_{0j}^{\max} \leq q_{opt} \leq q_{0j}^{\max} \end{cases}$$ (7)
This model does not take into consideration logistics related constraints, like transportation costs, capacity of transportation trucks, size of loading units, available warehouse capacity and the related outsourcing opportunities. Within the frame of this chapter, the mathematical model for the computation of the multi-item economic order quantity is described. It takes into consideration the above-mentioned constraints and numerical examples validate the model.

The objective function has four cost-based parts, like order cost, warehousing cost, purchasing price and material handling cost related to transportation:

\[ C = P + C_W + C_o + C_{MH} \rightarrow \min. \] (8)

The first part of the objective function is the purchasing price, which depends on the specific purchasing price and the purchasing quantity:

\[ P = \sum_{i=1}^{m} p(q_i) \cdot Q_i \] (9)

The second part of the objective function is the warehousing cost, which depends on the specific warehousing cost and the purchasing quantity:

\[ C_W = \sum_{i=1}^{m} c_w \cdot p(q_i) \cdot q_i \] (10)

where the specific warehousing cost is calculated as the multiplication of the purchasing price and the cost of capital per year in percent. The warehousing cost in this model cannot be calculated as in the case of the standard model, because the below-mentioned constraints have great impact on the solution and the optimal solution is limited by them:

\[ C_w = \sum_{i=1}^{m} \sqrt{q_i \cdot c_w \cdot p(q_i) \cdot \sigma_i} \] (11)

If the total order is more than the available warehouse capacity, then the warehousing cost be calculated as follows:

\[ \sum_{i=1}^{m} q_i > CAP_{WH} \rightarrow C_W = \sum_{i=1}^{m} q_i \cdot p(q_i) \cdot \sigma_i + \sum_{h \in \Psi} q_h \cdot p(q_h) \cdot \sigma_i \cdot \vartheta \] (12)

If \( \vartheta < 1 \) then the inventory holding cost in the outsourced warehouse is cheaper, than in the owned warehouse. If \( \vartheta > 1 \) then the inventory holding cost in the outsourced warehouse is more expensive, than in the owned warehouse.

The third part of the objective function is the total order cost, which is the sum of the order costs of products:

\[ C_o = \sum_{i=1}^{m} c_{oi} \cdot \frac{q_i}{q_i} \] (13)

The fourth part of the objective function is the total material handling cost related with transportation, which includes the costs of transportation, packaging, loading, and unloading:

\[ C_{MH} = \sum_{i=1}^{m} c_{mhi} \cdot \frac{q_i}{q_i} \] (14)

The first constraint is the capacity of the warehouse. We can take into consideration the warehouse capacity as a hard constraint, if there is no available outsourcing possibility to store the ordered products:

\[ \sum_{i=1}^{m} q_i \leq CAP_{WH} \] (15)

and the optimal solution can be limited by this constraint as a soft constraint, if we can store the ordered products in outsourcing warehouses:

\[ \forall h \in \Psi: p(q_h) = p(q_h) \cdot \vartheta \] (16)

The second constraint is the available transportation capacity. We can define the upper limit of the available capacity of trucks and it is not allowed to exceed this upper limit

\[ \forall i: q_i \leq q_{max} \] (17)

The third constraint is the size of the available loading units, which can define fix lots available for orders:

\[ \forall i: \frac{q_i}{c_{LUI}} = \ln \left( \frac{q_i}{c_{LUI}} \right) \] (18)

Based on the above-mentioned model (see Eq 8-18) we can calculate the optimal purchasing portfolio based on economic order quantity method using heuristic optimization methods. We have used evolutive heuristics to solve the above-described model. Within the frame of the next chapter we will demonstrate the
computational results and describe the impact of transportation costs, capacity of transportation trucks, size of loading units, available warehouse capacity and the related outsourcing opportunities.

4. RESULTS AND DISCUSSION

The first scenario includes four products and four suppliers. The total demands, the specific order costs and the cost of capital are given. Each product has five quantity discount range, as shown in Table 2.

The computational results of optimal economic order quantity model using the conventional mathematical model described in Eq. 1-7 are shown in Fig. 1. The total cost of the inventory problem is 4,621,377 USD but transportation costs are not taken into consideration. If we take into consideration the transportation costs, then the economic order quantities are shifted, as shown in Fig. 2. Considering the transportation costs and added them to the costs of the computed cost regarding economic order quantity we can define the total cost of the purchasing within the time frame as 6,193,625 USD.

In the next, let us analyze the impact of the above-mentioned additional constraints, like size of loading units, available capacity of trucks, available capacity of warehouse and additional cost of outsourced warehousing if suitable. In this Scenario 2 the upper limit of warehouse capacity is 16,000 pcs, an outsourcing possibility for storage of products is available with an $\bar{\theta} = 2.3$ multiplier. The other additional parameters are shown in Table 3.

As Fig. 3 demonstrates, the warehouse, transportation and loading unit related constraints has significant impact on the solution. Using the mathematical model described with Eq. 8-18 we can integrate the constraints into the conventional economic order quantity model, and we can define the new purchasing portfolio using evolutive heuristic in Excel Solver. Considering the warehouse capacity constraints, the optimal purchasing portfolio gives a total cost of 5,599,398 USD which is 90.405% of the original total cost (see Scenario 2a in Fig. 3).

**Table 2. Input parameters of Scenario 1**

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<th>Parameters</th>
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<th>Product B</th>
<th>Product C</th>
<th>Product D</th>
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<td>TD</td>
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<td>CoC</td>
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<td>10501/20000</td>
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*TD=Total demand for the time window [pcs]. SOP=Specific order price [USD/order]. STC=Specific transportation cost [USD/order]. CoC=Cost of capital [%]. DR=Discount rate*

**Table 3. Additional input parameters of Scenario 2 regarding constraints**

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*SLU=Size of loading units [pcs/LU]. CT=Capacity of truck [pcs/truck]*
Fig. 1. Results of Scenario 1: conventional computation of economic order quantity with quantity discount

Fig. 2. Results of Scenario 1: conventional computation of economic order quantity with quantity discount and costs of transportation
Considering the warehouse capacity and an upper limit of transportation trucks as constraints, the optimal purchasing portfolio gives a total cost of 5,627,287 USD, which means, that the transportation-related constraints provoke an additional cost of 27,888 USD (see Scenario 2b in Fig. 3). Considering the warehouse capacity and the fixed size of loading units as constraints, the optimal purchasing portfolio gives a total cost of 5,616,378 USD, which means, that the fixed size of loading unit has a great impact on the total cost, influences the optimal order quantity and gives an additional cost of 16,979 USD (see Scenario 2c in Fig. 3). Considering all three constraints (the warehouse capacity, the capacity of trucks and the fixed size of loading units) as constraints, the optimal purchasing portfolio gives a total cost of 5,636,139 USD, which means, that the three constraints gives an additional cost of 36,770 USD (see Scenario 2d in Fig. 3). The new mathematical model takes into consideration the three constraints and the cost of the optimized purchasing portfolio is cheaper with 9% comparing to the conventional computation method of economic order quantity with quantity discounts.

5. CONCLUSION

The computation of economic order quantity is a well-known and widely used method of inventory management. There is a wide range of models, which consider various environments and constraints. Within the frame of this article the authors highlighted the fact that logistic processes has a great impact on optimal solution of inventory problems, where not only the purchasing price but also the costs and constraints of logistic related operations, like warehousing, transportation, building of loading units, packaging, collection or distribution must be considered. The added value of this paper is the modelling of an economic order quantity problem, where the available capacity of the warehouse, the available capacity of trucks for transportation of products from the suppliers to the production company and the size of loading unit as a constraint of lot size are considered. The computational results show that logistic operations and the costs of material handling processes must be taken into consideration while making decisions regarding the purchasing portfolio. Managerial decisions can be influenced...
by the results of this research because the above-described analysis makes it possible to support managerial decisions regarding inventory holding, warehousing and transportation strategies. The results of the above-mentioned model and method can be used in real-scenarios, especially in the case of batch ordering processes of independent products. As a future research direction, we would like to focus on the lean aspects of inventory management problems [29], especially in cyber-physical supply chain environment [30].

ACKNOWLEDGEMENTS

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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